Equations sheet for FYS1120

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Electric fields

Coulomb's law

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho}{r^2}\hat{\mathbf{r}}\, d\tau$$

Dipoles

$$\tau = \boldsymbol{\mu}\times\mathbf{B} \qquad \tau = \mathbf{p}\times\mathbf{E}$$

$$\boldsymbol{\mu} = I\mathbf{A} \qquad \mathbf{p} = q\mathbf{d}$$

$$U_B = -\boldsymbol{\mu}\cdot\mathbf{B} \qquad U_E = -\mathbf{p}\cdot\mathbf{E}$$

Potential, energy and work

$$W_{a\to b} = U_a - U_b$$

$$U = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

$$V_B - V_A = -\int_A^B \mathbf{E}\cdot\mathbf{dl}$$

$$\nabla V = -\mathbf{E}$$

Energy stored in magnetic and electric fields.

$$U = \frac{1}{2}\int\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)d\tau$$

Energy stored in solenoid and capacitor:

$$U_B = \frac{1}{2}LI^2, \quad U_E = \frac{1}{2}\frac{Q^2}{C}$$

Maxwell's equations

In general

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \oint_S \mathbf{E}\cdot \mathrm{d}\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \oint_S \mathbf{B}\cdot\mathrm{d}\mathbf{A} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \oint_L \mathbf{E}\cdot\mathrm{dl} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{A}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$$

$$\oint_L \mathbf{B}\cdot\mathrm{dl} = \mu_0 I_S + \mu_0\epsilon_0\frac{d}{dt}\int_S \mathbf{E}\cdot d\mathbf{A}$$

In matter

$$\nabla\cdot\mathbf{D} = \rho_f \qquad \oint_S \mathbf{D}\cdot\mathrm{d}\mathbf{A} = Q_{f_{enc}}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \oint_S \mathbf{B}\cdot\mathrm{d}\mathbf{A} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \oint_L \mathbf{E}\cdot\mathrm{dl} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{A}$$

$$\nabla\times\mathbf{H} = \mathbf{J}_f + \frac{\partial\mathbf{D}}{\partial t} \qquad \oint_L \mathbf{H}\cdot\mathrm{dl} = I_{f_{enc}} + \frac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{A}$$

Definitions

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

In linear media

$$\mathbf{P} = \epsilon_0\chi_e\mathbf{E} \quad \mathbf{D} = \epsilon\mathbf{E}$$

$$\mathbf{M} = \chi_m\mathbf{H} \quad \mathbf{H} = \frac{1}{\mu}\mathbf{B}$$

Magnetism

$$\Phi = \int_S \mathbf{B}\cdot d\mathbf{S}$$

Field inside infinitely long solenoid:

$$\mathbf{B} = \mu_0 NI$$

Field between two coaxial cylinders:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r}$$

Field inside the smallest of two coaxial cylinders:

$$\mathbf{B} = \frac{\mu_0 I r}{2\pi a^2}$$

Field outside infinitely long conducting wire:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r}$$

Faraday's law and emf

$$\epsilon = \int \mathbf{f_s} \cdot d\mathbf{l}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

$$\mathbf{J} = \sigma \mathbf{f} = \sigma \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right)$$

Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{q\mathbf{v} \times \mathbf{r}}{r^2}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times \hat{\mathbf{r}}}{r^2} dl = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$$

Mutual inductance

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

$$\mathcal{E}_2 = -M \frac{di_1}{dt}$$

$$\mathcal{E}_1 = -M \frac{di_2}{dt}$$

Continuity of magnetic flux

$$\oint_S \mu_0 \mathbf{H} \cdot d\mathbf{A} = \mathbf{0} \, .$$

over a closed surface.

Circuits

Effect:

$$P = VI \, .$$

Over ohmic resistance:

$$P = RI^2 = \frac{V^2}{R} \, .$$

Voltage drop over capacitor:

$$V = \frac{q}{C}$$

Charging capacitor in RC-circuit:

$$q = \mathcal{E} C \left[1 - e^{(-t/(RC))} \right]$$

Decharging capacitor:

$$q = Q_0 e^{(-t/(RC))}$$

The time constant τ of RC circuit expresses how fast the RC-circuit is charged or discharged:

$$\tau = RC$$

Self inductance:

$$L = \frac{N\Phi_B}{i} \Longleftrightarrow \mathcal{E} = -L \frac{di}{dt}$$

Units

Henry:

$$\mathrm{H} = \frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^2 \cdot A^2}} = \frac{\mathrm{J}}{\mathrm{A^2}} = \frac{\mathrm{Wb}}{\mathrm{A}} = \frac{\mathrm{V \cdot s}}{\mathrm{A}} \quad (1)$$

$$= \frac{\mathrm{J/C \cdot s}}{\mathrm{C/s}} = \frac{\mathrm{J \cdot s^2}}{\mathrm{C^2}} = \frac{\mathrm{m^2 \cdot kg}}{\mathrm{C^2}} = \Omega \cdot \mathrm{s} \quad (2)$$

Ampere:

$$\mathrm{A} = \frac{\mathrm{C}}{\mathrm{s}}$$

Tesla:

$$\mathrm{T} = \frac{\mathrm{V \cdot s}}{\mathrm{m^2}} = \frac{\mathrm{N}}{\mathrm{A \cdot m}} = \frac{\mathrm{Wb}}{\mathrm{m^2}} = \frac{\mathrm{kg}}{\mathrm{C \cdot s}} = \frac{\mathrm{kg}}{\mathrm{A \cdot s^2}}$$

1 Constants

Proton mass

$$m_p = 1.67 \times 10^{-27} \, \mathrm{kg}$$

Proton charge

$$q_p = 1e = 1.602 \times 10^{-19} \, \mathrm{C}$$

Electron mass

$$m_e = 9.11 \times 10^{-31} \, \mathrm{kg}$$

Electron charge

$$q_e = -1e = -1.602 \times 10^{-19} \, \mathrm{C}$$

Electrical permittivity in vacuum

$$\epsilon_0 = 8.85 \times 10^{-12} \, \mathrm{C^2 N^{-1} m^{-2}}$$

Gravitational constant

$$G = 6.67 \times 10^{-11} \, \mathrm{m^2 kg^{-1} s^{-2}}$$